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STRESS-DEFORMED STATE OF A SEMI-INFINITE ICE SHEET UNDER THE ACTION OF A MOVING LOAD

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The behavior of an infinite ice sheet upon motion thereon of a load limited in plan has been considered in [1-8] and other studies. Practice interest in the stress deformed state of an ice sheet in the presence of a free edge along which a pressure system moves for the purpose of ice breaking has arisen relatively recently. This is related to the introduction of resonant ice breaking methods realized with amphibious air cushion vessels (ACV's) [2].

Experiments under model and natural conditions (Fig. 1) have noted significant changes in the ice-breaking capabilities of such craft in the presence of an open area of water and corresponding vessel maneuvering. The experiments showed a doubling of the ice thickness breakable in operations carried out in the immediate vicinity of an edge. However all the advantages of such ice-breaking technology and complete principles of ice behavior under such loading conditions remain to be described.

The present study will consider the stress-deformed state of a semi-infinite ice sheet under the action of a moving ACV. The problem will be solved numerically.

Problem formulation and theoretical solution. We choose as the main equation that of the viscoelastic ice oscillations under the influence of a point force P moving with velocity v:

$$\frac{Gh^3}{3}\left(1+\tau_\Phi\frac{\partial}{\partial t}\right)\nabla^4 w+\rho_w g w+\rho_\ell h\frac{\partial^2 w}{\partial t^2}+\rho_w\frac{\partial \Phi}{\partial t}\Big|_{z=0} = P\delta(x-vt,\ y-0). \tag{1}$$

Here G is the shear modulus of the ice elasticity; h, the ice sheet thickness; w, the deflection of the ice; ρ_i and ρ_w, the densities of ice and water; g, acceleration of gravity; Φ, the liquid motion potential, satisfying the boundary conditions and the Laplace equation; δ, a delta-function; τ_Φ, the deformation relaxation time.

The axes x and y lie in the plane of the ice sheet, with the x-axis along the direction of load motion, and the z-axis directed upward.

The liquid motion potential satisfies the boundary conditions:

$$\frac{\partial \Phi}{\partial z}\Big|_{z=-H} = 0; \tag{2}$$

$$\frac{\partial w}{\partial t}-\frac{\partial \Phi}{\partial z}\Big|_{z=0} = 0 \tag{3}$$

(where H is the basin depth).

Numerical solution can be simplified by some preliminary transformations of the original equations.

We satisfy Eq. (2), writing Φ in the form

$$\Phi = \varphi(x,y,t)\operatorname{ch}k(z+H),\ k = \text{const}. \tag{4}$$

Substituting Eq. (4) into Eq. (3) and the Laplace equation

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2} = 0,$$

we obtain

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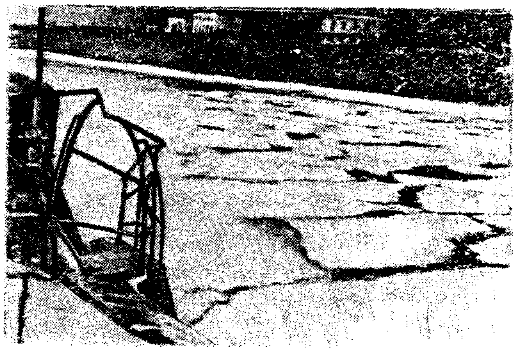

Fig. 1

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + k^2\varphi = 0; \tag{5}$$

$$\frac{\partial w}{\partial t} - \varphi k \mathrm{sh} kH = 0. \tag{6}$$

Transforming to a moving coordinate system $\xi = x - vt$, $\tau = t$, we reduce Eq. (1) to the form

$$\frac{Gh^3}{3}\nabla^4 w + \tau_\Phi \frac{Gh^3}{3}\left(\frac{\partial}{\partial\tau} - v\frac{\partial}{\partial\xi}\right)\nabla^4 w + \rho_w g w + \rho_\ell h\left(\frac{\partial^2}{\partial\tau^2} - 2v\frac{\partial^2}{\partial\tau\partial\xi} + v^2\frac{\partial^2}{\partial\xi^2}\right) w + \rho_w\left(\frac{\partial\Phi}{\partial\tau} - v\frac{\partial\Phi}{\partial\xi}\right)\Big|_{z=0} = P\delta(\xi, y = 0). \tag{7}$$

We write w as $w = w_1 + w_2$, and Φ as $\Phi = \Phi_1 + \Phi_2$, where w_1 and Φ_1 are independent of τ. Then

$$\frac{Gh^3}{3}\nabla^4 w_1 - \tau_\Phi v\frac{Gh^3}{3}\frac{\partial}{\partial\xi}\nabla^4 w_1 + \rho_w g w_1 + \rho_\ell h v^2\frac{\partial^2 w_1}{\partial\xi^2} - \rho_w v\frac{\partial\Phi_1}{\partial\xi}\Big|_{z=0} = P\delta(\xi, y = 0), \quad \frac{Gh^3}{3}\left(1 + \tau_\Phi\left(\frac{\partial}{\partial\tau} - v\frac{\partial}{\partial\xi}\right)\right)\nabla^4 w_2 + \rho_w g w_2 + \rho_w h\left(\frac{\partial^2}{\partial\tau^2} - 2v\frac{\partial^2}{\partial\tau\partial\xi} + v^2\frac{\partial^2}{\partial\xi^2}\right) w_2 + \rho_w\left(\frac{\partial\Phi_2}{\partial\tau} - v\frac{\partial\Phi_2}{\partial\xi}\right)\Big|_{z=0} = 0 \tag{8}$$

or in the former variables

$$\frac{Gh^3}{3}\left(1 + \tau_\Phi\frac{\partial}{\partial t}\right)\nabla^4 w_2 + \rho_w g w_2 + \rho_\ell h\frac{\partial^2 w_2}{\partial t^2} + \rho_w\frac{\partial\Phi_2}{\partial t}\Big|_{z=0} = 0 \tag{9}$$

Equation (8) describes the steady state problem, Eq. (9), the natural ice oscillations about the surface specified by Eq. (8), and will not be considered further.

After transformation to the moving coordinate system Eqs. (5), (6) take on the form

$$\frac{\partial^2\varphi}{\partial\xi^2} + \frac{\partial^2\varphi}{\partial y^2} + k^2\varphi = 0; \tag{10}$$

$$\frac{\partial w}{\partial\tau} - v\frac{\partial w}{\partial\xi} - \varphi k \mathrm{sh} kH = 0. \tag{11}$$

Considering that $\Phi = \Phi_1 + \Phi_2$, (Φ_1 being independent of τ), we also take

$$\varphi = \varphi_1(\xi, y) + \varphi_2(\xi, y, \tau).$$

Then, for w_1, φ_1, we obtain from Eqs. (10), (11)

$$\frac{\partial^2\varphi_1}{\partial\xi^2} + \frac{\partial^2\varphi_1}{\partial y^2} + k^2\varphi_1 = 0; \tag{12}$$

$$v\frac{\partial w_1}{\partial\xi} + \varphi_1 k \mathrm{sh} kH = 0. \tag{13}$$

Since $\Phi_1 = \varphi_1(\xi, y)\text{ch}k(z + H)$ it then follows from Eq. (4) that from Eqs. (8), (12), (13) we have

$$\frac{Gh^3}{3}\nabla^4 w_1 - \tau_\phi \upsilon \frac{Gh^3}{3}\frac{\partial}{\partial\xi}\nabla^4 w_1 + \rho_w g w_1 + \rho_\ell h \upsilon^2 \frac{\partial^2 w_1}{\partial\xi^2} - \rho_w \upsilon \frac{\partial\varphi_1}{\partial\xi}\text{ch}kH = P\delta\ (\xi, y = 0),$$
$$\frac{\partial^2\varphi_1}{\partial\xi^2} + \frac{\partial^2\varphi_1}{\partial y^2} + k^2\varphi_1 = 0,\ \upsilon\frac{\partial w_1}{\partial\xi} + \varphi_1 k\text{sh}kH = 0. \tag{14}$$

Eliminating φ_1 from Eq. (14), we arrive at

$$\frac{Gh^3}{3}\nabla^4 w_1 + \tau_\phi \upsilon \frac{Gh^3}{3}\frac{\partial}{\partial\xi}\nabla^4 w_1 + \rho_w g w_1 + \left(\rho_\ell h + \frac{\rho_w}{k}\text{cth}kH\right)\upsilon^2 \frac{\partial^2 w_1}{\partial\xi^2} = P\delta(\xi, y = 0),$$
$$\frac{\partial^3 w_1}{\partial\xi^3} + \frac{\partial^3 w_1}{\partial\xi\partial y^2} + k^2\frac{\partial w_1}{\partial\xi} = 0. \tag{15}$$

It is difficult to solve Eq. (15) analytically, so a numerical determination of w_1 will be expedient.

To do this we use the finite element method. We consider a limited, but sufficiently large region of the ice field and adjacent open water, on the boundary of which we may consider displacements equal to zero. The dimensions of the region are selected in solution of each concrete problem. To construct the discrete model we use a rectangular finite element with 16 degrees of freedom

$$w_1(\xi, y) = \sum_{i=1}^{16} N_i(\xi, y) q_i, \tag{16}$$

where $N_i(\xi, y)$ are form functions and q_i are lattice point displacements.

Substituting Eq. (16) in Eq. (15) and using the Bubnov–Galerkin method, we arrive at the system of equations

$$[K]\{q\} = \{P\}, ([T] - k^2[S])\{q\} = 0. \tag{17}$$

Here {q} is the lattice point displacement vector; {P}, the external load vector; [K], [T], [S], matrices formed of the corresponding individual finite element matrices.

We write {q} as a linear combination

$$\{q\} = \sum_{i=1}^{n} C_i\{q\}_i, \tag{18}$$

where $\{q_i\}$ is the eigenvector of the matrix $[T] - k^2[S]$, corresponding to the eigenvalue k_i^2 and n is the number f node displacements. Then

$$\sum_{i=1}^{n} [K]_i\{q\}_i C_i = \{P\}. \tag{19}$$

Here the matrix $[K]_i$ corresponds to the eigenvalue k_i^2.

Determining the unknown constants C_i from system (19) and substituting them in Eq. (18), we obtain the solution.

Results. To study the dependence of the stress-deformed state of a semi-infinite ice sheet upon vessel speed and distance of the line of motion from the edge of the ice, a series of calculations were made for various values of these parameters. Motion of a load with $P = 0.4 \cdot 10^6$ N at velocities in the range 0-16 m/sec was considered, with the line of motion parallel to the edge varied from 50 m within the ice to 50 m away in the open water.

The calculation results showed that with increase in load velocity from zero, deflections and stresses initially decreased from their static values. This can be explained by the effect of water inertia forces. With development of the oscillatory process which occurs with further increase in velocity, the deflections and stresses increase and reach maximum values at the resonant load speed. We note that similar stress-deformed state parameters were found for the case of an infinite ice sheet, with resonant velocity values practically coinciding for the two cases.

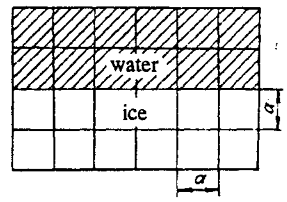

Fig. 2

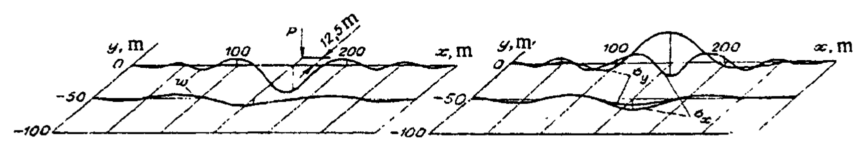

Fig. 3

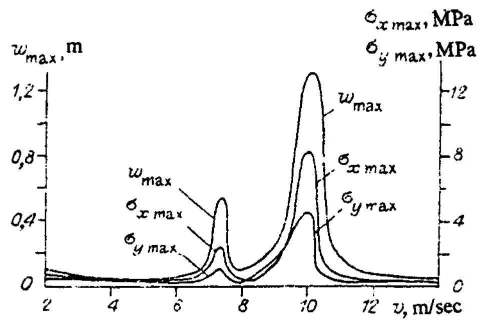

Fig. 4

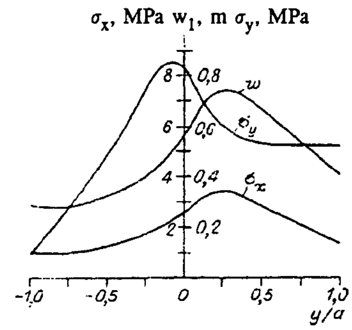

Fig. 5

Analysis of the effect of the distance of the line of motion from the ice edge allows the following conclusions. As was expected, upon approach of the load to the edge from the direction of the ice itself, the values of w, σ_x, σ_y increase. While w and σ_x increase 2-3 times, σ_y increases 7 times. Further translation of the load from the ice edge with motion along that edge in open water leads to further increase in w and σ_x with decrease in σ_y. This can be explained by the fact that motion along the edge in the open water excites purely gravitational waves. The water medium is more yielding, so the total energy of these waves proves to be higher than that of the deflection-gravitation waves excited by the load during motion over the ice itself. The gravitational waves penetrating under the ice sheet lead to a redistribution of elastic state reactions and increase in deformations.

An increase in deflections and stresses is observed out to some certain distance of the line of motion from the ice sheet edge, after which for further departure of the load those values decrease asymptotically.

Calculations were performed by the finite element method. The discrete model of the ice–water system is shown in Fig. 2, where a = 50 m. As is evident from Fig. 2, the region had the form of a 300 × 200 m rectangle, divided into 24 finite elements 50 × 50 m in size. Experience from previous studies permits us to be confident that for the given number of elements and cell size the values found are quite reliable.

A general picture of the character of stress-deformed state of semi-infinite ice sheet canbe gatheredfrom Fig. 3, which shows spatial distributions of w, σ_x, σ_y from an ice thickness h = 0.25 m, basin depth H = 5 m, and velocity v = 4 m/sec.

The calculation results were used to construct graphs of the maximum deflection w_{max} and maximum stresses σ_{max}, σ_{max} as functions of velocity v (Fig. 4) and of deflection and stress at the midpoint of the edge as functions of the position of the vessel motion line (Fig. 5).

CONCLUSIONS

The presence of a free edge, along which the load moves, has practically no effect on the resonant velocity values v_p obtained for an infinite ice sheet of finite thickness (for the case calculated v_r = 7.3 m/sec. The second peak in amplitude and stress (Fig. 4) at v = 10 m/sec, just as in the theory of wave resistance of a basin to open water, exists only in theory.

To break an ice sheet over a large area the most effective AVC motion is not along the edge of the ice, but in the open water at a certain distance from the edge (as is evident from Fig. 5, for the case calculated this distance is 12.5 m). This conclusion follows from overall analysis of the stress-deformed state of the ice sheet [4]. The moving deflection-gravity waves developed in the ice have a long front and a direction perpendicular to the motion of the load, and at the peaks of these waves σ_x causes the main failure of the ice. Additional damage (further breakup of fragments, splitting off of the edge, etc.) occurs upon transformation of these waves at the breakage points.

If the problem is to widen a channel extending through the ice, then the ACV should move at the resonant velocity along the edge of the ice sheet. The stress σ_y is then a maximum and the open region expands by breakoff of the channel edges.

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